

MAS231

Assignment 8

Geometry II
Due on 31 March 2008

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the Green Post-Box on the ground floor in the Maths building before 16:00 on Monday.

1. (a) Prove that $\frac{1}{2}\text{trace}(F_I^{-1}F_{II}) = \frac{LG-2MF+NE}{2(EG-F^2)}$.

(b) Prove that if $t_i = a_i\sigma_u + b_i\sigma_v$, $i = 1, 2$, then $t_1 \cdot t_2 = v_1^T F_I v_2$ where $v_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$.

2. Consider the surface patch

$$\sigma(u, v) = (u, v, u^2 + v^3),$$

$$U = \{(u, v) : -\infty < u < \infty, -\infty < v < \infty\}$$

covering a surface S .

(a) Compute the first fundamental form F_I and second fundamental form F_{II} for this surface patch.

(b) Find K_G for this surface patch and show that surface S is elliptic where $v > 0$, hyperbolic where $v < 0$, and parabolic for $v = 0$.

3. Consider the cone with surface patch $\sigma(u, v) = (u, v, \sqrt{u^2 + v^2})$, $U = \{(u, v) : u^2 + v^2 > 0\}$.

(a) Compute the second fundamental form F_{II} .

(b) Find the Gauss curvature K_G and mean curvature H for this surface patch.

(c) Compute the principle curvatures K_1 , K_2 from K_G and H .

4. (a) State Euler's theorem about principal vectors.

(b) For the cone in the previous question, find the unit principal tangent vector t_1 at the point $\sigma(u, v)$ for $K_1 = 0$.

(c) Find a curve on the cone through the point $p = \sigma(u, v)$ with tangent t_1 there. What can you say about its normal curvature K_n at p ?